FAR BEYOND

# MAT122

## Quotient Rule

Stony Brook University

# Review

Power Rule

$$(ax^n)' = \frac{d}{dx}ax^n = nax^{n-1}$$

$$\left(e^x\right)' = e^x$$

Product Rule

$$[f \cdot g]' = f'g + fg'$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

Do: find $\left(5\sqrt{x^3}\right)'$

write answer as a single fraction containing a radical

Do: find $\frac{d}{dx}\left(\sqrt{x}\, e^x\right)$

write first term as single fraction

# Quotient Rule Formula

When two *differentiable* functions are divided, use the **Quotient Rule** to take derivative:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad \text{where } g \neq 0$$

$g$ not $g$ prime

ex. Find $y'$ if $y = \dfrac{x^2 + x - 2}{x^3 + 6}$

$f = x^2 + x - 2 \qquad g = x^3 + 6$

$f' = 2x + 1 \qquad g' = 3x^2$

Do: both FOILs

$$y' = \frac{f'g - fg'}{g^2} = \frac{(2x+1)(x^3+6) - (x^2+x-2)(3x^2)}{(x^3+6)^2}$$

for quotient rule, simplify numerator
leave denominator FACTORED

Do: distribute

$$= \frac{2x^4 + 12x + x^3 + 6 - \left(3x^4 + 3x^3 - 6x^2\right)}{(x^3+6)^2}$$

Do: combine like terms

$$= \frac{2x^4 + 12x + x^3 + 6 - 3x^4 - 3x^3 + 6x^2}{(x^3+6)^2} = \boxed{\frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}}$$

# Quotient Rule – cont'd

ex. Find the equation of the tangent line to the curve $y = \dfrac{e^x}{1+x^2}$ at $\left(1, \dfrac{e}{2}\right)$.

$$f = e^x \qquad g = 1 + x^2$$
$$f' = e^x \qquad g' = 2x$$

$$m = \frac{dy}{dx} = \frac{e^x(1+x^2) - e^x(2x)}{(1+x^2)^2}$$ simplify numerator

$\dfrac{d}{dx}(y)$

$$= \frac{e^x + x^2e^x - 2xe^x}{(1+x^2)^2}$$

$$m\Big|_{\left(1,\frac{e}{2}\right)} = \frac{e^1 + 1^2e^1 - 2\cdot 1\cdot e^1}{(1+1^2)^2} = \frac{e+e-2e}{(1+1)^2} = \frac{0}{4} = 0$$

$$\boxed{y - y_1 = m(x - x_1)}$$

$$y - \frac{e}{2} = \cancel{0(x-1)}\ 0$$

$$\boxed{y = \frac{e}{2}}$$

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

# Quotient Rule or Not?

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Sometimes it's easier to simplify a fraction rather than use quotient rule:

ex. differentiate $f(x) = \dfrac{3x^2 + 2\sqrt{x}}{x}$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

**Method #1 - quotient rule**

$f = \boxed{3x^2 + 2\sqrt{x}}$

$f' = 6x + \cancel{2}\cdot\dfrac{1}{\cancel{2}\sqrt{x}} = \boxed{6x + \dfrac{1}{\sqrt{x}}}$

$\boxed{\begin{array}{l} g = x \\ g' = 1 \end{array}}$

$$f'(x) = \frac{\left(6x + \frac{1}{\sqrt{x}}\right)x - \left(3x^2 + 2\sqrt{x}\right)(1)}{x^2}$$

distribute

$$= \frac{\overset{3x^2}{\cancel{6x^2}} + \cancel{\sqrt{x}} - \cancel{3x^2} \overset{-\sqrt{x}}{-\cancel{2\sqrt{x}}}}{x^2}$$

$$\frac{x}{\sqrt{x}} = \frac{x^1}{x^{\frac{1}{2}}} = x^{\frac{1}{2}}$$

$$= \frac{3x^2}{x^2} - \frac{x^{1/2}}{x^2}$$

write as separate fractions

$$= 3 - x^{-\frac{3}{2}}$$

**Method #2 - power rule**

$$f(x) = \frac{3x^2}{x} + \frac{2x^{\frac{1}{2}}}{x}$$

$$= 3x + 2x^{-\frac{1}{2}}$$

$$f'(x) = 3 - \cancel{2}\cdot\frac{1}{\cancel{2}}\,x^{-\frac{3}{2}}$$

$$= 3 - x^{-\frac{3}{2}}$$

$$\boxed{f'(x) = 3 - \frac{1}{\sqrt{x^3}}}$$

# Using Product AND Quotient Rules

ex. differentiate $f(x)=\dfrac{1-xe^x}{x+e^x}$

$$f'(x)=\frac{\left(-e^x-xe^x\right)\left(x+e^x\right)-\left(1-xe^x\right)\left(1+e^x\right)}{\left(x+e^x\right)^2}$$

$$=\frac{-xe^x-e^{2x}-x^2e^x-xe^{2x}-(1+e^x-xe^x-xe^{2x})}{\left(x+e^x\right)^2}$$

$$=\frac{-\cancel{xe^x}-e^{2x}-x^2e^x-\cancel{xe^{2x}}-1-e^x+\cancel{xe^x}+\cancel{xe^{2x}}}{\left(x+e^x\right)^2}$$

$$=\boxed{\frac{-e^{2x}-x^2e^x-1-e^x}{\left(x+e^x\right)^2}}$$

product rule ☺

$$f=1-xe^x$$

$u=x \qquad v=e^x$

$u'=1 \qquad v'=e^x$

$$f'=0-\underbrace{(e^x+xe^x)}_{u'v+uv'}$$

$$=-e^x-xe^x$$

$$g=x+e^x$$

$$g'=1+e^x$$

$$e^x\cdot e^x=\left(e^x\right)^2$$

$$=e^{2x}$$

$$\boxed{(f\cdot g)'=f'\cdot g+f\cdot g'}$$

$$\boxed{\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}}$$